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All-pay war ☆

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ABSTRACT

We study a model of conflicts and wars in which the outcome is uncertain not because of luck on the battlefield as in standard models, but because countries lack information about their opponent. In this model expected resource levels and production and military technologies are common knowledge, but realized resource levels are private information. Each country decides how to allocate its resources to production and warfare. The country with the stronger military wins and receives aggregate production. In equilibrium both comparative and absolute advantages matter: a larger resource share is allocated to warfare by the country with a comparative advantage in warfare at relatively low realized resource levels, and by the country with an absolute disadvantage in warfare at relatively high realized resource levels. From an ex-ante perspective the country with a comparative advantage in warfare is more likely to win the war unless its military potential is much lower.

1. Introduction

The outcome of many conflicts and wars is uncertain from the perspective of the involved parties or countries, as well as from the outsiders' perspective. Standard models of conflicts and wars account for this uncertainty by assuming that luck plays a crucial role on the battlefield. In this paper we study an alternative model in which countries are uncertain about the outcome not because of luck on the battlefield, but because they lack information about their opponent and, consequently, its endogenous military power. We imagine that throughout history rivalling tribes, possibly from remote forests or mountainous areas, were imperfectly informed about each other's resources. But even nowadays, countries often lack precise estimates of their opponent's labor-force and stock of human and physical capital and, consequently, its productive and military power.[1] Moreover, even countries that may know the size of their opponent's labor-force during peacetime may not know how many people who are normally out of the labor-force are willing and able to help out on the home front (i.e., in

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[1] One reason is that official figures on, e.g., labor supply and production are often biased, and that the size of these biases is typically unknown. Shleifer and Treisman (2005) argue that official figures tend to overestimate true resources and production in communist countries in which managers routinely inflate production figures. In contrast, official figures may underestimate true resources and production in capitalist countries in which individuals and businesses may want to evade taxation.

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production) or the battlefield during wartime.[2] Similarly, countries may lack accurate information about how dedicated and motivated their opponent's people are to use their labor and capital for the best of their country during wartime.[3]

In our model there are two countries (or regions) characterized by their production and military technologies, and the expected level of their resources. These characteristics may differ across countries and are common knowledge. Each country, however, only knows its own realized resource level, and it can choose how to allocate these resources to production and warfare. The resource allocation and the technologies determine domestic production and military power. The country with the greater military power wins and can consume all goods that have been produced in the two countries, while the loosing country gets nothing.

We characterize monotone continuous equilibrium strategies for all possible values of the parameters representing the countries' production and military technologies and their expected resource levels. Interestingly, these strategies depend on comparative as well as absolute advantages in warfare.[4] They are straightforward if the country with a comparative advantage in warfare has a large absolute disadvantage in warfare. For any realized resource level this country then allocates all resources to warfare, while its opponent only allocates some fraction of its resources to warfare. From an ex ante perspective, the opponent is nevertheless more likely to win the war because of its higher military potential.

Equilibrium strategies are more involved if the country with a comparative advantage in warfare has also an absolute advantage or only a modest absolute disadvantage in warfare. This country then allocates all resources to warfare up to some threshold and follows a non-decreasing and concave strategy for higher resource levels. Its opponent allocates a constant fraction of its resources to warfare up to some threshold and follows an increasing non-linear strategy for higher resource level. Hence, at relatively low resource levels it is again the country with a comparative advantage in warfare that allocates a higher share of its resources to warfare. However, at relatively high resource levels absolute advantages matter: the country with an absolute advantage in warfare allocates a smaller share of its resources to warfare in order to avoid diverting many more resources away from production when already winning the war with high probability. From an ex ante perspective, the country with a comparative advantage in warfare is nevertheless more likely to win the war.

The theoretical literature on conflicts and wars contains two main strands. The first looks at reasons why conflicts emerge, and the second studies how conflicts are fought.[5] Our model contributes to the second strand. It is closely related to the standard models of conflicts and wars that go back to Haavelmo (1954) and have been popularized by Garfinkel (1990), Grossman (1991), Hirshleifer (1991, 2001), and Skaperdas (1992). Garfinkel and Skaperdas (2007, Section 3.2) present a synthesis of these models, which typically have four key features: First, there is a war taking place for exogenous reasons. Second, each country can choose how to allocate its resources to production and warfare. Third, the mapping from the resources that the different countries allocate to warfare to the outcome of the war is probabilistic, often modeled using a Tullock (1980) contest success function. Fourth, the winning country can consume all production. While keeping the first two and the last of these features, we assume that the country with the stronger military wins for sure.[6] Moreover, we add the assumption that countries are imperfectly informed about their opponent's resources. Our model thus offers a complementary view according to which countries are uncertain about the outcome of the war not because of luck on the battlefield, but because they lack information about their opponent.

Despite these differences in the setup, our results share some properties with the standard models of conflicts and wars: Countries with too few resources may face a binding resource constraint and allocate all their resources to warfare, and countries with a comparative advantage in warfare tend to allocate more resources to warfare than their opponent. However, there are some noteworthy differences: In the standard models the countries' resource allocation and their payoffs are in equilibrium independent of the distribution of the resources between the two countries as long as resource constraints are not binding in either of the countries (Garfinkel and Skaperdas, 2007). Therefore, if the countries have the same technologies and differ only with respect to their resource endowment, the resource poorer country fully compensates its disadvantage by allocating a higher share of its resources to warfare. As a result both countries win the war with the same probability and have the same expected payoff regardless of the resource distribution. This result, which Hirshleifer (1991) famously dubbed the Paradox of Power (in its strong form), is probably the best known result of the theoretical literature on conflicts and wars. In our model the countries' equilibrium strategies increase in their own realized resource level and are not just a simple choice of a particular resource allocation.[7] Therefore, equilibrium payoffs depend on the distribution of the resources between the two countries in general, and not only when one of the countries faces a binding resource constraint (as in the standard models). In particular, it holds that if the two countries have the same technologies and the same expected resource level, then the country with the lower realized resource level allocates fewer resources to warfare than

[2] For example, many were surprised by the dramatic increase in women's labor-force participation in the United States during World War II.

[3] As a recent example, many were surprised by the (initial) reluctance of Iraqis to fight when the United States and its allies invaded Iraq to overthrow the regime of Saddam Hussein. Many were also surprised by the fierce resistance of some Iraqi factions in later years.

[4] The country with the higher ratio of military to production technology has a comparative advantage in warfare. Further we say that the country that would have the stronger military when allocating its expected resource level to warfare has an absolute advantage in warfare.

[5] Garfinkel and Skaperdas (2007) and Blattman and Miguel (2010) review the literature. See also Jackson and Morelli (2011) for a survey of the first strand of this literature.

[6] A harmless form of war in which the larger army won for sure was practiced by the German *Landesknechte* at the end of the sixteenth century: when two opposing parties met, the respective number of soldiers were counted, and the side with the lower number surrendered (Hochheimer, 1967, p. 83).

[7] This follows because the strategy space is a subset of $\mathbb{R}_+$ in the standard models, but a subset of functions from $\mathbb{R}_+$ to $\mathbb{R}_+$ in our model.

its opponent. Hence it loses for sure and ends up getting nothing. This result contrasts with the Paradox of Power obtained in the standard models.

Another interesting difference between the implications of our model and the standard models arises when production and military technologies differ across countries. In the standard models only comparative advantages matter, at least, if resource constraints are not binding, while in our model both comparative and absolute advantages matter. As discussed above, we find that comparative advantages matter if the realized resource levels are relatively low, while absolute advantages matter if these levels are relatively high. Moreover, the countries' equilibrium strategies depend also on expected resource levels. This, in turn, allows us to study the role of expectations, which is impossible in standard models in which resource endowments are common knowledge. We find that countries fight more aggressively when they were expected to be resource poor ex ante, but turn out to be relatively resource rich ex post than they fight in the opposite case.

Most other models of conflicts in which countries have some private information contribute to the first strand of the theoretical literature on conflicts and wars by studying the emergence of conflicts. Thereby they typically take military power as given (e.g., Fearon, 1995). Building on these models, Meirowitz and Sartori (2008) present a model with a similar flavor as ours in that war can occur between two countries that have invested in military power but cannot observe each other's investment. In their model private information follows from countries playing mixed strategies when deciding how much to invest in military power.[8] The complexity and generality of their model come however at the cost that equilibrium strategies cannot be derived.

As we model war as an asymmetric auction with incomplete information, our paper also relates to the literature on auction theory. Our model thereby shares some features with both all-pay and winner-pay auctions. Like in all-pay auctions, all bids need to be paid, i.e., no resources allocated to warfare can be used to produce consumption goods. Hence our model has some similarities with models of all-pay auctions with incomplete information, which go back to Amann and Leininger (1996) and Krishna and Morgan (1997).[9] However the payoff structure and the players' problems are quite different in our model than in all-pay auctions. First, the winner's payoff depends on the loser's bid in our model, but not in all-pay auctions. Second, the loser's payoff is zero independently of his bid in our model, but depends on his bid in all-pay auctions. This feature that the loser's payoff is always zero may suggest that our model is actually more closely related to winner-pay auctions. From all the contributions on winner-pay auctions, our paper might be closest to Che and Gale (1998) who assume incomplete information and financially constrained bidders. However a major difference between our model and winner-pay auctions is that the winner's payoff decreases not only in his own bid, but also in the loser's bid.[10]

The contributions of Fey (2008) and Adamo and Matros (2009) introduce incomplete information into an otherwise standard symmetric Tullock rent seeking contest and an otherwise standard symmetric Blotto game, respectively. Hence, our model is related to these contributions as it also presents a contest with incomplete information. Despite this similarity there are substantial differences between our paper and these contributions: Our paper differs from Fey (2008) in that incomplete information is about resource endowments rather than bidding costs; that the player with the higher effective bid (i.e., the stronger military) wins for sure rather than with some probability given by a Tullock contest success function; and that the players can use their resources to bid or to produce the winner's prize, which is thus endogenous and decreasing in the bids of both players. Similarly, our paper differs from Adamo and Matros (2009) in that there is only one contest but with an endogenous prize in our model, while there are several contests with fixed prizes in a Blotto game. The players' resource allocation problems and the trade-offs they face are thus quite different. Our model further differs from Fey (2008) and Adamo and Matros (2009) by allowing for asymmetries in the players' technologies. It is noteworthy that our model allows for a closed-form solution in the whole parameter space despite these asymmetries.

The remainder of the paper is organized as follows: Section 2 introduces the model. Section 3 presents some preliminary results. Section 4 derives and discusses the equilibrium. Section 5 concludes.

2. The model

There are two countries that are at war for some exogenous reason. Each country $i = 1, 2$ is characterized by three parameters: its military technology $\tilde{\lambda}_i \in \mathbb{R}_+$, its production technology $\tilde{\beta}_i \in \mathbb{R}_+$, and the expected resource level $R_i \in \mathbb{R}_+$. These parameters are common knowledge. Each country's actual resource endowment is $2r_iR_i$, where r_i is independently and identically drawn from the uniform distribution on $[0, 1]$. (Essentially, country i's actual resource endowment is thus

[8] Jackson and Morelli (2009) study a model similar to Meirowitz and Sartori (2008), but assume that investments in military power are observable.

[9] Feess et al. (2008) study an all-pay auction with incomplete information in which a player may have a handicap in a similar way as a country may have a lower military technology in our model.

[10] To better understand the similarities and differences between our model and winner-pay auctions notice that the winner's payoff in our model equals aggregate production when all resources were devoted to production minus the aggregate amount of production that is foregone due to fighting. The former, i.e., the size of the total available pie, is analogous to the valuation of the auction winner in the interdependent value paradigm as it is determined by the private signals of all players. Yet, different from the auction models, the latter, i.e., the slices foregone, depends not only on the winner's strategy but also on the loser's.

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drawn from a uniform distribution on $[0, 2R_i]$.) The realizations r_1 and r_2 are private information while their distribution is common knowledge.[11,12]

Each country acts as a single player, and the two countries must simultaneously decide how to allocate their resource endowment to production and warfare. Given the realization r_i, country i chooses an allocation b_i such that $2b_iR_i$ and $2(r_i - b_i)R_i$ represent the resources allocated to warfare and production, respectively. The resource constraint requires $b_i \in [0, r_i]$.

We define $\lambda_i \equiv 2\tilde{\lambda}_i R_i$ and $\beta_i \equiv 2\tilde{\beta}_i R_i$, which are country i's maximal possible military power and its maximal possible production, respectively. We also call λ_i country i's military potential, and β_i its production potential. The actual military power of country i is represented by $\lambda_i b_i$, and its actual production of consumption goods by $\beta_i(r_i - b_i)$. The outcome of the war is deterministic in that the country with the higher military power $\lambda_i b_i$ wins for sure. The winning country can consume all goods that have been produced in the two countries. Therefore, given choices b_i and b_j, and realizations r_i and r_j, the payoff of country i is

$$\tilde{u}_i(b_i, b_j; r_i, r_j) = \begin{cases} 0 & \text{for } \lambda_i b_i < \lambda_j b_j, \\ \beta_i(r_i - b_i) & \text{for } \lambda_i b_i = \lambda_j b_j, \\ \beta_1(r_1 - b_1) + \beta_2(r_2 - b_2) & \text{for } \lambda_i b_i > \lambda_j b_j. \end{cases}$$

In this game, the strategy space is such that country i's strategies are of the form $b_i = f_i(r_i) : [0, 1] \to [0, r_i]$. We look for a Bayesian Nash equilibrium in monotone continuous strategies that are differentiable almost everywhere.

We further define $\beta \equiv \frac{\beta_1}{\beta_2}$ and $\lambda \equiv \frac{\lambda_1}{\lambda_2}$, and we assume without loss of generality that $\beta \leqslant \lambda$, which implies $\frac{\beta_1}{\lambda_1} \leqslant \frac{\beta_2}{\lambda_2}$ and, equivalently, $\frac{\tilde{\beta}_1}{\tilde{\lambda}_1} \leqslant \frac{\tilde{\beta}_2}{\tilde{\lambda}_2}$. That is, we call the country with a comparative advantage in warfare country 1, and the country with a comparative advantage in production country 2. We also say that the country with the higher military potential λ_i has an absolute advantage in warfare, as this country could ensure military victory whenever both drew the same r_i.[13] Country 1 thus has an absolute advantage in warfare if $\lambda > 1$, and an absolute disadvantage if $\lambda < 1$. Subsequently we refer to countries as players, thereby calling player 1 "she" and player 2 "he". Moreover, we call their choices of b_i their bids or *real bids*, while referring to $\lambda_i b_i$ as their *effective bids*. Effective bids play a key role in this game because the player with the higher effective bid wins the war.

3. Preliminary results

In this section we first present an important lemma. We then study a simplified version of the game introduced in the previous section to understand some of the main forces at work.

Lemma 1. *In any monotone equilibrium it holds that $f_1(0) = f_2(0) = 0$, that $f_1(.)$ and $f_2(.)$ are non-decreasing, and that $\lambda f_1(1) = f_2(1)$.*

Proof. It directly follows from the requirement $f_i(r_i) \in [0, r_i]$ that $f_1(0) = f_2(0) = 0$. Together with the required monotonicity of $f_i(r_i)$, this implies that $f_1(.)$ and $f_2(.)$ must be non-decreasing. We prove $\lambda f_1(1) = f_2(1)$ by contradiction. Suppose $\lambda_i f_i(1) > \lambda_j f_j(1)$. For $r_i = 1$, player i is then better off by deviating and playing $b_i = \frac{f_j(1)\lambda_j}{\lambda_i} < f_i(1)$, as this increases the winner's payoff while i still wins with probability one. Hence it must hold in any monotone equilibrium that $\lambda_i f_i(1) = \lambda_j f_j(1)$. $\square$

Lemma 1 already puts some structure on the players' bidding strategies. It directly follows from the resource constraint that players with zero resources cannot allocate any resources to warfare. As a consequence, monotone strategies must be non-decreasing. Moreover, no player ever bids more than necessary to win with probability one because the winner's payoff decreases in the resources he or she has allocated to warfare. Effective bids must thus coincide at the top, i.e., if $r_1 = r_2 = 1$.

We next solve our game assuming that the three properties specified in Lemma 1 hold, but abstracting from the resource constraints for $r_i > 0$ and $i = 1, 2$. This simplified version of the game has a closed-form solution that is easy to interpret

[11] As discussed in the Introduction, we think of resources as a composite measure that include a country's labor-force and its stock of human and physical capital as well as the motivation and dedication of the people to use their labor and capital effectively during wartime. For example, a very low realization r_i may thus represent a situation in which country i has far less labor and capital than its opponent expected; or one in which the people in country i lack the motivation and dedication to work and fight hard during wartime.

[12] Assuming that the lower bound of the resource distributions is zero simplifies the analysis, mainly because it leads almost directly to one of the boundary conditions (see Lemma 1). When assuming that the lower bound is strictly positive, we can still derive what we will call the quasi-equilibrium strategies, and these strategies also have the same properties as those derived in Section 3. The proper equilibrium strategies equal these quasi-equilibrium strategies in a symmetric setting in which countries have the same technologies and the same expected resource level. However, in an asymmetric setting deriving the proper equilibrium strategies becomes considerably more difficult if the lower bound of the resource distributions is strictly positive.

[13] Hence comparative advantages depend on production and military technologies, while absolute advantages depend on military technologies and expected resource levels.

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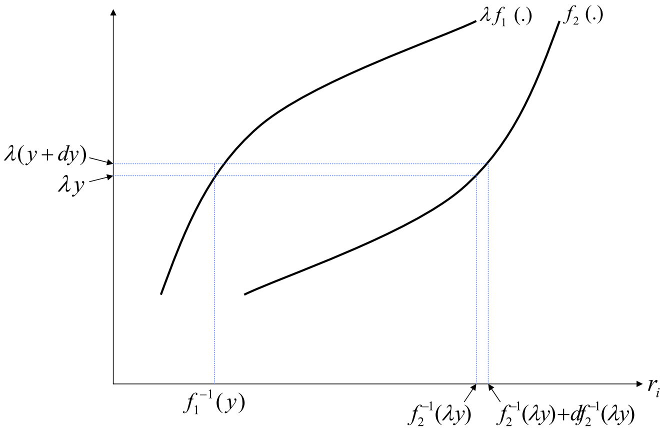

Fig. 1. Player 1's trade-off.

and helpful to understand how the players' incentives shape their behavior. To avoid confusion, we call the equilibrium of this simplified version of our game a quasi-equilibrium.

We start by focusing on the bidding strategy chosen by player 1 assuming that player 2 chooses the non-decreasing strategy $f_2(r_2)$. Player 1 wins if and only if she bids $y > \frac{f_2(r_2)}{\lambda}$, i.e., if and only if $r_2 < f_2^{-1}(\lambda y)$. Hence her expected payoff is

$$u_1(y; r_1) \equiv \int_0^1 \tilde{u}_1\big(y, f_2(r_2); r_1, r_2\big)\, dr_2 = \int_0^{f_2^{-1}(\lambda y)} \big[\beta_1(r_1 - y) + \beta_2\big(r_2 - f_2(r_2)\big)\big]\, dr_2. \tag{1}$$

She faces a trade-off as a marginal increase in y increases the probability of winning, but reduces the prize, i.e., aggregate production of consumption goods. It follows from the first-order condition that the optimal bid $y = f_1(r_1)$ must satisfy

$$-\beta f_2^{-1}(\lambda y) + \big[\beta\big(f_1^{-1}(y) - y\big) + f_2^{-1}(\lambda y) - \lambda y\big]\frac{df_2^{-1}(\lambda y)}{dy} = 0, \tag{2}$$

or, equivalently,

$$\big[\beta_1\big(f_1^{-1}(y) - y\big) + \beta_2\big(f_2^{-1}(\lambda y) - \lambda y\big)\big]\, df_2^{-1}(\lambda y) = \beta_1\, dy\, f_2^{-1}(\lambda y). \tag{3}$$

Condition (3) and Fig. 1 illustrate the trade-off that player 1 faces. Consider a type of player 1 that bids y and thinks about bidding $y + dy$. The benefit from increasing the bid by dy occurs if this increase turns her into a winner. This event occurs with probability $df_2^{-1}(\lambda y)$ and generates an expected marginal benefit as represented on the left-hand side of (3). The marginal cost of increasing the bid is borne if player 1 is already a winner when bidding y. This event occurs with probability $f_2^{-1}(\lambda y)$. The opportunity cost of increasing the bid is the forgone production $\beta_1\, dy$. Hence the right-hand side of (3) represents the expected marginal cost of increasing the bid.

Similarly, if player 1 chooses the non-decreasing strategy $f_1(r_1)$, then player 2's optimal bid $y = f_2(r_2)$ must satisfy

$$-f_1^{-1}(y) + \big[\beta\big(f_1^{-1}(y) - y\big) + f_2^{-1}(\lambda y) - \lambda y\big]\frac{df_1^{-1}(y)}{\lambda\, dy} = 0. \tag{4}$$

It follows from the system of the two differential equations (2) and (4):

Lemma 2. *Disregarding any constraints, the players' strategies are mutual best responses if they are of the form*

$$f_1(r_1) = \frac{\beta}{\beta + 2\lambda} r_1 + \frac{1}{2\beta + \lambda} K_0 r_1^{\frac{\beta}{\lambda}} + K_1 r_1^{-(1+\frac{\beta}{\lambda})}, \tag{5}$$

$$f_2(r_2) = \frac{\lambda}{2\beta + \lambda} r_2 + \frac{\lambda\beta}{\beta + 2\lambda} K_0^{-\frac{\lambda}{\beta}} r_2^{\frac{\lambda}{\beta}} + K_2 r_2^{-(1+\frac{\lambda}{\beta})}, \tag{6}$$

where K_0, K_1 and K_2 are constants.

Proof. See Appendix A. □

Lemmas 1 and 2 imply that the quasi-equilibrium strategies must be of form (5) and (6), respectively, and satisfy the boundary conditions $f_1(0) = f_2(0) = 0$ and $\lambda f_1(1) = f_2(1)$. It follows:

Corollary 1. *The players' quasi-equilibrium strategies are*

$$f_1(r_1) = \frac{\beta}{\beta + 2\lambda} r_1 + \frac{1}{2\beta + \lambda} r_1^{\frac{\beta}{\lambda}}, \tag{7}$$

$$f_2(r_2) = \frac{\lambda}{2\beta + \lambda} r_2 + \frac{\lambda\beta}{\beta + 2\lambda} r_2^{\frac{\lambda}{\beta}}. \tag{8}$$

Proof. Eqs. (5) and (6) satisfy $f_1(0) = f_2(0) = 0$ only if $K_1 = K_2 = 0$, and then $\lambda f_1(1) = f_2(1)$ only if $K_0 = 1$. Inserting $K_0 = 1$ and $K_1 = K_2 = 0$ into (5) and (6) gives (7) and (8). □

The quasi-equilibrium strategies are increasing. Moreover, they are linear if $\beta = \lambda$, i.e., if none of the players has a comparative advantage in warfare, and they even coincide if $\beta = \lambda = 1$. Therefore, the stronger player with the higher realized resource level $2r_i R_i$ always wins in a symmetric setting in which both players have the same technologies $\tilde{\beta}_i$ and $\tilde{\lambda}_i$, and the same expected resource level R_i. This result contrasts with the Paradox of Power obtained in the standard models of conflicts and wars.

In case of asymmetries leading to $\beta < \lambda$, player 1's quasi-equilibrium strategy is strictly concave, and player 2's quasi-equilibrium strategy strictly convex. Since $\lambda f_1(0) = f_2(0)$ and $\lambda f_1(1) = f_2(1)$, it follows that $\lambda f_1(r) > f_2(r)$ for all $r \in (0, 1)$. That is, in the absence of resource constraints, player 1 who has a comparative advantage in warfare chooses the higher effective bid, i.e., the stronger military, for any realization $r \in (0, 1)$. Player 1 thus wins the war when r_1 is higher or only slightly lower than r_2. From an ex ante perspective, i.e., in expectation before nature draws r_1 and r_2, player 1 is therefore more likely to win the war than her opponent.

Turning from effective to real bids, it directly follows from $\lambda f_1(r) > f_2(r)$ for all $r \in (0, 1)$ that $f_1(r) > f_2(r)$ for all $r \in (0, 1)$ if $\lambda \leqslant 1$. Hence player 1 chooses a higher real bid and allocates a higher share of her resources to warfare than her opponent for any realization r when she has a comparative advantage, but an absolute disadvantage in warfare. This is necessary for her to build the stronger military. However, if $\lambda > 1$, there exists a unique threshold $\hat{r} \in (0, 1)$ such that $f_1(r) > f_2(r)$ for $r \in (0, \hat{r})$ and $f_1(r) < f_2(r)$ for $r \in (\hat{r}, 1)$.[14] That is, if player 1 has a comparative and an absolute advantage in warfare, she allocates a higher share of her resources to warfare than her opponent does when both countries' realized resource levels are low relative to their expected resource levels R_i, but a lower share when their realized resource levels are relatively high. The former is driven by her incentive to specialize in warfare, and the latter by her incentive not to allocate many more resources to warfare when already winning the war with high probability.

To further study how expected and realized resource levels affect the players' quasi-equilibrium strategies, we rewrite (7) and (8) as

$$2R_1 f_1(r_1) = \frac{\tilde{\beta}}{\tilde{\beta} + 2\tilde{\lambda}}(2r_1 R_1) + \frac{1}{2\tilde{\beta} + \tilde{\lambda}} \frac{2R_2}{(2R_1)^{\frac{\tilde{\beta}}{\tilde{\lambda}}}} (2r_1 R_1)^{\frac{\tilde{\beta}}{\tilde{\lambda}}}, \tag{9}$$

$$2R_2 f_2(r_2) = \frac{\tilde{\lambda}}{2\tilde{\beta} + \tilde{\lambda}}(2r_2 R_2) + \frac{\tilde{\lambda}\tilde{\beta}}{\tilde{\beta} + 2\tilde{\lambda}} \frac{2R_1}{(2R_2)^{\frac{\tilde{\lambda}}{\tilde{\beta}}}} (2r_2 R_2)^{\frac{\tilde{\lambda}}{\tilde{\beta}}}, \tag{10}$$

where the left-hand sides denote the resources allocated to warfare by players 1 and 2, respectively, and where $\tilde{\beta} \equiv \frac{\tilde{\beta}_1}{\tilde{\beta}_2}$ and $\tilde{\lambda} \equiv \frac{\tilde{\lambda}_1}{\tilde{\lambda}_2}$. The following results are worth emphasizing: First, the resources allocated to warfare by player $i = 1, 2$ increase in its own realized resource level $2r_i R_i$ and in the expected resource level R_j of its opponent $j \neq i$. Second, for any realized resource level $2r_i R_i$, player i allocates more resources to warfare if R_i is small and r_i large than if R_i is large and r_i small. Hence players fight more aggressively if they were expected to be resource poor ex ante, but turn out to be relatively resource rich ex post than in the opposite case. Third, the distribution of resources between the two players matters. To see this, let us keep R_1, R_2 and $2(r_1 R_1 + r_2 R_2)$ fixed for the moment. An increase in r_i (and the associated decrease in r_j) then motivates player i to allocate more resources to warfare and player j to allocate fewer resources to warfare. As a consequence, the expected payoff of both players changes. This result, which obtains even though we still abstract from the players' resource constraints, is in contrast to the standard models' result that the players' resource allocation is independent of the distribution of resources between the players if resource constraints are not binding.

[14] Existence and uniqueness of this threshold can be established using the following observations. First, $f_1(r_1)$ is continuously increasing and concave, while $f_2(r_2)$ is continuously increasing and convex. Second, $f_1(0) = f_2(0)$ and $\lim_{r \to 0^+} f_1'(r) > \lim_{r \to 0^+} f_2'(r)$ since $\beta < \lambda$. Third, $f_1(1) < f_2(1)$ since $\lambda f_1(1) = f_2(1)$ and $\lambda > 1$.

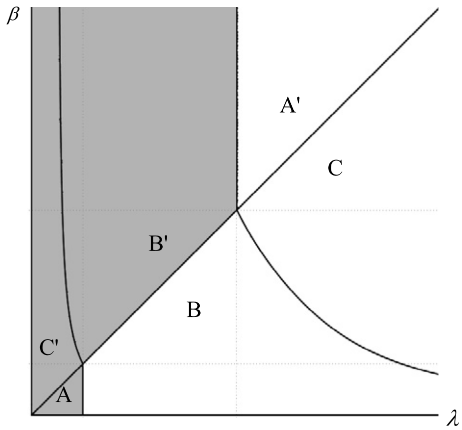

Fig. 2. Regions in the parameter space.

The quasi-equilibrium strategies (7) and (8) characterize equilibrium behavior if and only if they satisfy the resource constraints $f_i(r_i) \leqslant r_i$ for $r_i > 0$ and $i = 1, 2$. This is the case if and only if $\beta = \lambda \in [\frac{1}{2}, 2]$. If $\beta = \lambda \notin [\frac{1}{2}, 2]$, the quasi-equilibrium strategy of the player with lower β_i and λ_i violates the resource constraint for all realizations r_i. And if $\beta < \lambda$, player 1's quasi-equilibrium strategy and any other strategy of form (5) violate the resource constraint for r_1 sufficiently close to zero.[15]

4. Equilibrium

In this section we first characterize the players' equilibrium strategies for all possible values of β and λ. We then compare their real and effective bids. The general pattern will be similar as in the quasi-equilibrium. The behavioral differences that will occur are due to the resource constraints that the players are facing, and not due to changes in their incentives. The insights that we have gained in the previous section are therefore helpful to understand equilibrium behavior.

We know from the previous section that strategies satisfying (5) and (6) are mutual best responses, and that they are non-linear unless $\beta = \lambda$. Also we know that any strategy of type (5) violates player 1's resource constraint for r_1 sufficiently close to zero if $\beta < \lambda$. We thus conjecture that player 1's equilibrium strategy includes bidding all resources for realizations r_1 up to some threshold $c_l > 0$, and possibly to follow a non-linear strategy of type (5) for $r_1 > c_l$. The following result will therefore be useful:

Lemma 3. *Suppose player 1 follows a non-decreasing strategy with $f_1(r_1) = r_1$ for $r_1 \leqslant c_l$. Then player 2's best response that is lower than λc_l is $f_2(r_2) = \frac{r_2}{2}$.*

Suppose player 2 follows a non-decreasing strategy with $f_2(r_2) = \frac{r_2}{2}$ for $r_2 \leqslant 2\lambda c_l$. Then player 1's best response is $f_1(r_1) = r_1$ for $r_1 \leqslant c_l$.

Proof. Given player 1's strategy characterized in the first statement, player 2's best response lower than λc_l follows from inserting $f_1(r_1) = r_1$ into condition (4), which then reduces to $f_2^{-1}(\lambda y) = 2\lambda y$, implying $f_2(r_2) = \frac{r_2}{2}$. Given player 2's strategy characterized in the second statement, it follows that $\frac{\partial u_1(y;r_1)}{\partial y} = 2\lambda[\beta_1(r_1 - 2y) + \beta_2 \lambda y]$ for $r_1 \leqslant c_l$, which is positive since $y \leqslant r_1$ and $\beta \leqslant \lambda$. Hence it is optimal for player 1 to bid all resources whenever $r_1 \leqslant c_l$. □

We next derive the equilibrium strategies separately for different regions of the parameter space. These regions are shown in Fig. 2.[16] We focus on regions A, B and C, which are consistent with our assumption $\beta \leqslant \lambda$. We do not explicitly derive equilibrium strategies for regions A′, B′ and C′ in which $\beta > \lambda$. However it is straightforward to show that these equilibrium strategies are symmetrical to those in regions A, B and C, respectively.

Region A is defined by $\beta \leqslant \lambda \leqslant \frac{1}{2}$. Hence player 1 has a comparative advantage but a large absolute disadvantage in warfare. She has little incentive to allocate resources to production because she can produce relatively little anyway, and because she needs to bid much more aggressively than her opponent if she ever wants to win the war. We can thus explain equilibrium behavior using Lemmas 1 and 3 only.

[15] Note that $\lim_{r_1 \to 0^+} f_1'(r_1) = \infty$ if $f_1(r_1)$ is characterized by (5) and $\beta < \lambda$. Since $f_1(0) = 0$, it follows that $f_1(r_1) > r_1$ for $r_1 \to 0^+$.

[16] There are six parameters in our model ($\tilde{\beta}_i$, $\tilde{\lambda}_i$ and R_i for $i = 1, 2$), but equilibrium strategies are the same for all parameter constellations that lead to the same λ and β. Therefore, all parameter constellation can be represented in the (λ, β)-space.

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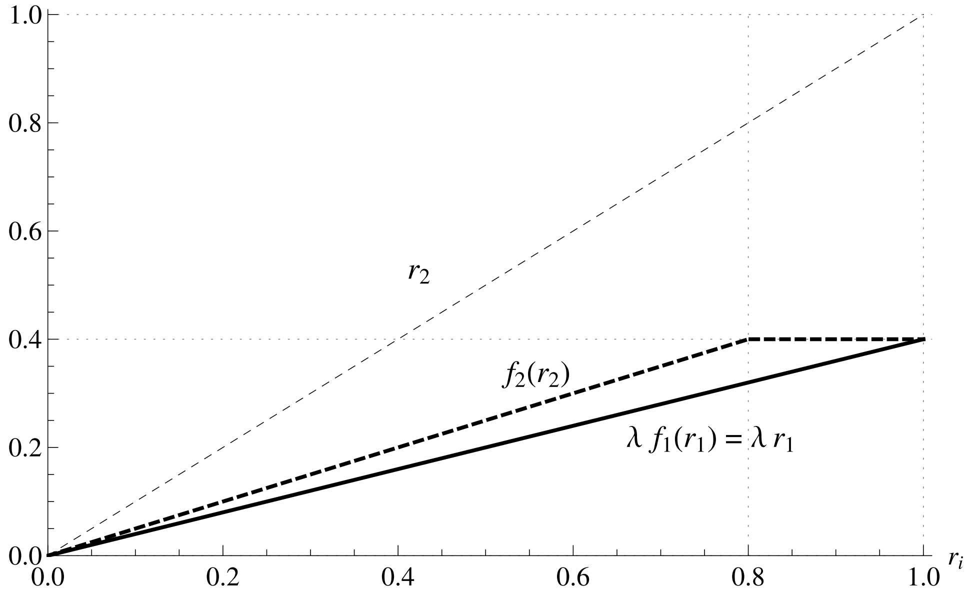

Fig. 3. Effective equilibrium bids in region A (with $\beta = 0.3$ and $\lambda = 0.4$).

Proposition 1. *Assume $\lambda \leqslant \frac{1}{2}$. Then player 1's equilibrium strategy is $f_1(r_1) = r_1$, and player 2's equilibrium strategy is $f_2(r_2) = \min\{\frac{r_2}{2}, \lambda\}$.*

Proof. It follows from Lemma 3 that $f_1(r_1) = r_1$ is player 1's best response. It follows from Lemma 3 that $f_2(r_2) = \frac{r_2}{2}$ is player 2's best response for $r_2 \in [0, 2\lambda]$, and from Lemma 1 and $f_1(1) = 1$ that player 2 should bid $f_2(r_2) \leqslant \lambda$ for all r_2. Hence player 2's best response is $f_2(r_2) = \min\{\frac{r_2}{2}, \lambda\}$. □

Fig. 3 illustrates the equilibrium strategies described in Proposition 1. Note that the proposition states our results in real bids, while the figure shows effective bids. Player 1 bids all resources for any realization r_1. Player 2's best response is to bid half his resources, but never more than necessary to win with probability one. Fig. 3 further shows that $\lambda f_1(r) \leqslant f_2(r)$ for all $r \in [0, 1]$ despite $f_1(r) \geqslant 2 f_2(r)$ for all $r \in [0, 1]$. We will come back to comparisons of the players' real and effective bids after characterizing the equilibrium strategies for $\lambda > \frac{1}{2}$.

The strategies described in Proposition 1 cannot explain equilibrium behavior when $\lambda > \frac{1}{2}$, as player 1 would have an incentive to deviate and to allocate some resources to production for $r_1 > \frac{1}{2\lambda}$. Nevertheless she still has an incentive to bid all resources for low r_1. To derive the equilibrium strategies we therefore use Lemmas 1 and 3 as well as Lemma 2. In particular, we conjecture that player 1's equilibrium strategy is $f_1(r_1) = r_1$ for $r_1 \in [0, c_l]$, where $c_l \in (0, 1)$, and of type (5) for $r_1 \in (c_l, 1]$, and that player 2's equilibrium strategy is $f_2(r_2) = \frac{r_2}{2}$ for $r_2 \in [0, 2\lambda c_l]$ and of type (6) for $r_2 \in (2\lambda c_l, 1]$. Given these conjectured equilibrium strategies, the system of Eqs. (5) and (6) must satisfy the boundary condition

$$\lambda f_1(c_l) = f_2(2\lambda c_l) = \lambda c_l. \tag{11}$$

It can then be shown:

Lemma 4. *Suppose player 1 follows a non-decreasing strategy with*

$$f_1(r_1) = c_l h\left(\frac{r_1}{c_l}\right) \tag{12}$$

for $r_1 > c_l$, where $c_l \in (0, \max\{1, \frac{1}{2\lambda}\})$ and $h(x) \equiv \frac{\beta}{\beta+2\lambda}x + \frac{2\lambda}{2\beta+\lambda}x^{\frac{\beta}{\lambda}} + \frac{2\lambda(\beta-\lambda)}{(\beta+2\lambda)(2\beta+\lambda)}x^{-(1+\frac{\beta}{\lambda})}$. Then player 2's best response that is higher than λc_l is

$$f_2(r_2) = \lambda c_l h\left(\left(\frac{r_2}{2\lambda c_l}\right)^{\frac{\lambda}{\beta}}\right). \tag{13}$$

Suppose player 2 follows a non-decreasing strategy with $f_2(r_2)$ given by (13) for $r_2 \geqslant 2\lambda c_l$. Then player 1's best response $f_1(r_1)$ that is higher than c_l is given by (12). It holds that $f_1'(.) > 0$, $f_1'(c_l) = 1$, $f_1''(.) < 0$ and $f_2'(.) > 0$.

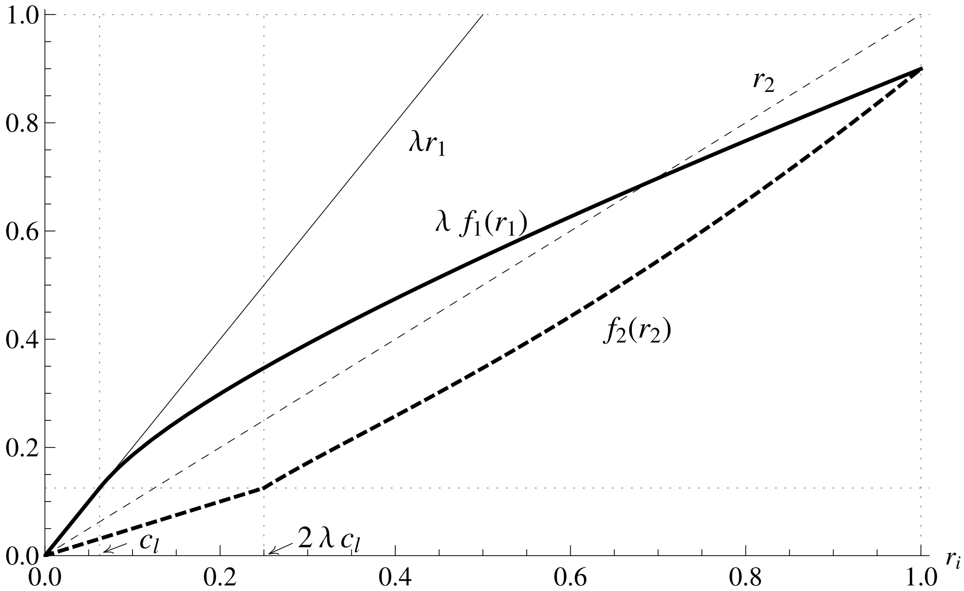

Fig. 4. Effective equilibrium bids in region B (with $\beta = 1$ and $\lambda = 2$).

Proof. See Appendix A. □

It is straightforward to see that the conjectured equilibrium strategies do not violate the players' resource constraints for $r_1 \leqslant c_l$ and $r_2 \leqslant 2\lambda c_l$. Also player 1's conjectured equilibrium strategy does not violate her resource constraint for any $r_1 > c_l$, as her strategy described by (12) satisfies $f_1'(c_l) = 1$ and is concave for $r_1 > c_l$. However it is a priori unclear whether or not player 2's conjectured equilibrium strategy violates the resource constraint for some $r_2 > 2\lambda c_l$. We know from Lemma 1 that the strategies described by (12) and (13) must satisfy the boundary condition $\lambda f_1(1) = f_2(1)$ if $f_2(r_2)$ does not violate the resource constraint for any $r_2 > 2\lambda c_l$. This boundary condition and (12) and (13) imply $c_l = (2\lambda)^{\frac{\lambda}{\beta-\lambda}}$. An equilibrium of the type conjectured therefore exists if and only if the strategy described by (13) satisfies $f_2(r_2) \leqslant r_2$ for all $r_2 > 2\lambda c_l$ when $c_l = (2\lambda)^{\frac{\lambda}{\beta-\lambda}}$. The following proposition establishes that this is the case if and only if

$$\lambda \leqslant \Lambda(\beta,\lambda) \equiv \left[(2\lambda)^{\frac{\lambda}{\beta-\lambda}} h\left((2\lambda)^{\frac{\lambda}{\lambda-\beta}}\right)\right]^{-1}, \tag{14}$$

and that $\Lambda(\beta,\lambda) > 2$ if $\max\{\beta, \frac{1}{2}\} < \lambda \leqslant \Lambda(\beta,\lambda)$. This proposition thus applies to region B, which is characterized by $\beta \leqslant \lambda$ and $\frac{1}{2} < \lambda \leqslant \Lambda(\beta,\lambda)$.

Proposition 2. *Assume $\frac{1}{2} < \lambda \leqslant \Lambda(\beta,\lambda)$, which implies $\Lambda(\beta,\lambda) > 2$. Then player 1's equilibrium strategy is $f_1(r_1) = r_1$ for $r_1 \in [0, c_l]$ and as described by (12) for $r_1 \in (c_l, 1]$, with $c_l = (2\lambda)^{\frac{\lambda}{\beta-\lambda}} < 1$. Player 2's equilibrium strategy is $f_2(r_2) = \frac{r_2}{2}$ for $r_2 \in [0, 2\lambda c_l]$ and as described by (13) for $r_2 \in (2\lambda c_l, 1]$, with $2\lambda c_l < 1$.*

Proof. See Appendix A. □

Fig. 4 illustrates the equilibrium strategies described in Proposition 2. It shows that player 1's resource constraint is binding for $r_1 \leqslant c_l$, while player 2 responds by bidding half his resources for $r_2 \leqslant 2\lambda c_l$. For higher realized resource levels, both players' strategies are non-linear and their effective bids coincide at the top.

It remains to explain equilibrium behavior in region C, which is characterized by $\beta \leqslant \lambda$ and $\lambda > \Lambda(\beta,\lambda)$. We know from the definition of $\Lambda(\beta,\lambda)$ that player 2's resource constraint must be binding at the top in this region, which of course affects player 1's strategy at high r_1. We conjecture that in this case the strategy profile satisfies the boundary condition

$$\lambda f_1(c_h) = f_2(1) = 1, \tag{15}$$

where $c_h < 1$. It then follows from Lemma 1 that player 1 bids $f_1(r_1) = \frac{1}{\lambda}$ for all $r_1 \geqslant c_h$. Therefore:

Proposition 3. *Assume $\lambda > \Lambda(\beta,\lambda)$. Then player 1's equilibrium strategy is $f_1(r_1) = r_1$ for $r_1 \in [0, c_l]$, as described by (12) for $r_1 \in (c_l, c_h]$, and $f_1(r_1) = \frac{1}{\lambda}$ for $r_1 \in (c_h, 1]$, with c_l being unique and implicitly defined by $c_l = (\lambda h((2\lambda c_l)^{-\frac{\lambda}{\beta}}))^{-1}$ and with*

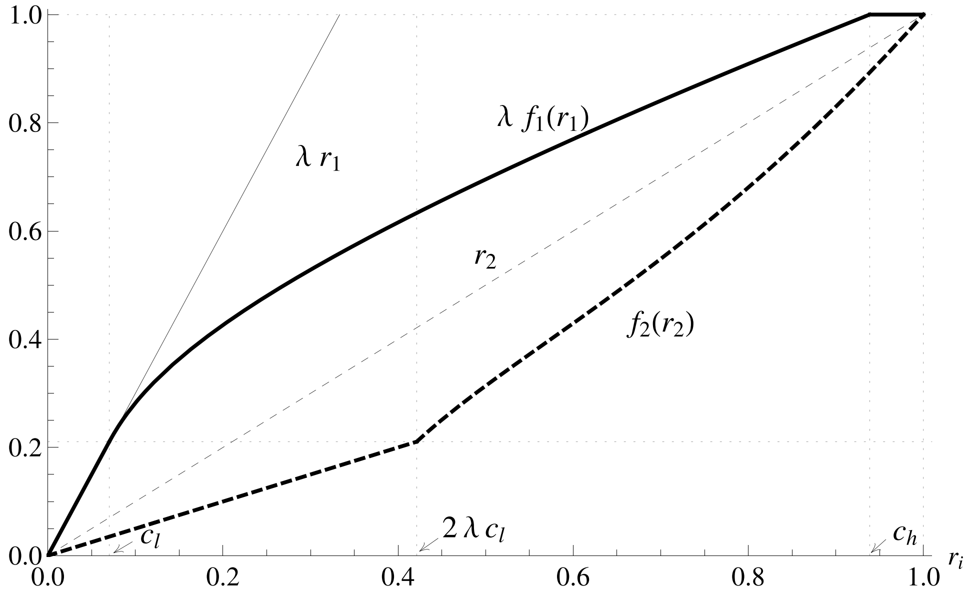

Fig. 5. Effective equilibrium bids in region C (with $\beta = 1$ and $\lambda = 3$).

$c_h = (2\lambda)^{-\frac{\lambda}{\beta}} c_l^{\frac{\beta-\lambda}{\beta}}$ *satisfying* $c_l < c_h < 1$. *Player 2's equilibrium strategy is* $f_2(r_2) = \frac{r_2}{2}$ *for* $r_2 \in [0, 2\lambda c_l]$ *and as described by* (13) *for* $r_2 \in (2\lambda c_l, 1]$, *with* $2\lambda c_l < 1$.

Proof. See Appendix A. □

Fig. 5 illustrates the equilibrium strategies described in Proposition 3. Unlike in Fig. 4, player 2's resource constraint is now binding at the top, and player 1 responds by never submitting any bid higher than necessary to win the war with probability one. Proposition 3 and Fig. 5 highlight that despite his comparative disadvantage in warfare, player 2 fights very aggressively and bids all his resources at high realizations r_2 when also having a large absolute disadvantage in warfare. Player 1 responds by bidding sufficiently aggressively to ensure her victory and the consumption of all her produced goods with probability one for a whole range of high realizations r_1.

Having derived the players' equilibrium strategies for all possible values of β and λ, we next compare their real and effective bids. We start by looking at the case in which $\beta = \lambda$ such that no player has a comparative advantage in warfare:

Proposition 4. *Assume* $\beta = \lambda$. *In equilibrium it then holds for all* $r \in (0, 1)$ *that* $f_1(r) > f_2(r)$ *if* $\lambda < 1$, $f_1(r) = f_2(r)$ *if* $\lambda = 1$, *and* $f_1(r) < f_2(r)$ *if* $\lambda > 1$; *and that* $\lambda f_1(r) < f_2(r)$ *if* $\lambda < \frac{1}{2}$, $\lambda f_1(r) = f_2(r)$ *if* $\lambda \in [\frac{1}{2}, 2]$, *and* $\lambda f_1(r) > f_2(r)$ *if* $\lambda > 2$.

Proof. Results for $\lambda \in [\frac{1}{2}, 2]$ directly follow from Corollary 1 and our discussion thereafter. Results for $\lambda < \frac{1}{2}$ directly follow from Proposition 1. Results for $\lambda > 2$ also follow from Proposition 1 after renaming player 1 as player 2, and vice versa. □

Proposition 4 states that the weaker player with lower production and military potential chooses higher real bids $f_i(r)$ for any realization $r \in (0, 1)$, just as in the quasi-equilibrium. As long as $\lambda \in [\frac{1}{2}, 2]$, allocating a higher share of his (or her) resources to warfare allows this player to compensate for his lower military potential λ_i and to end up with the same effective bid $\lambda_i f_i(r)$ for any $r \in (0, 1)$. However if his military potential λ_i is less than half as high as the opponent's, i.e., if $\lambda < \frac{1}{2}$ or $\lambda > 2$, then this weaker player ends up with the lower effective bid for any $r \in (0, 1)$. This result, which did not obtain in the quasi-equilibrium, is not due to the weaker player not wanting to bid more to compensate for his low military potential, but due to his resource constraint. As Proposition 1 implies, this player bids all of his resources, but this is not enough to reach the same effective bid as the stronger opponent who generally bids half of her resource endowment, but never more than necessary to win with probability one. From an ex ante perspective the two players are thus equally likely to win the war unless their military (and production) potential is sufficiently dissimilar.

We next compare real and effective bids for the case in which $\beta < \lambda$, such that player 1 has a comparative advantage in warfare.

Proposition 5. *Assume* $\beta < \lambda$. *In equilibrium it then holds that* $f_1(r) > f_2(r)$ *for all* $r \in (0, 1)$ *if* $\lambda \leqslant 1$. *Otherwise,* $f_1(r) > f_2(r)$ *for* r *below or sufficiently close to* $2\lambda c_l$, *and* $f_1(r) < f_2(r)$ *for* r *sufficiently close to* 1. *Further it holds for all* $r \in (0, 1)$ *that* $\lambda f_1(r) < f_2(r)$ *if* $\lambda < \frac{1}{2}$, $\lambda f_1(r) = f_2(r)$ *if* $\lambda = \frac{1}{2}$, *and* $\lambda f_1(r) > f_2(r)$ *if* $\lambda > \frac{1}{2}$.

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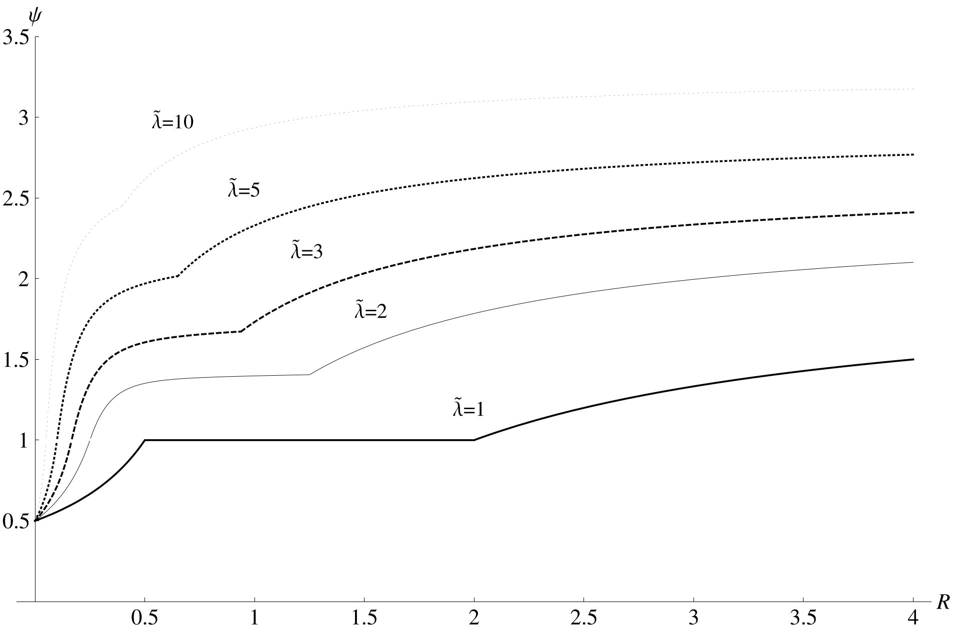

Fig. 6. Relative expected effective bids as a function of R.

Proof. See Appendix A. □

It follows from Proposition 5 that results relating to the players' real bids are again similar in equilibrium as in the quasi-equilibrium discussed in Section 3. If player 1 has a comparative advantage, but an absolute disadvantage in warfare, then she allocates for any realization r a higher share of her resources to warfare than her opponent. But if she has a comparative as well as an absolute advantage in warfare, then she allocates a higher share of her resources to warfare than her opponent if their realized resource levels are both small relative to their expected resource levels R_i, but a lower share than her opponent when their realized resource levels are both relatively high.

Proposition 5 also states (and Figs. 4 and 5 illustrate) that player 1 chooses the higher effective bid than her opponent for any realization r if her military potential λ_i is at least half as good as her opponent's. As argued earlier, player 1 has this incentive to build a stronger military because of her comparative advantage in warfare. But, for any r, player 1 ends up with the weaker military if her military potential is not even half as high as her opponent's. The reason for this result, which did not obtain in the quasi-equilibrium, is not that player 1 does not want to choose a higher effective bid, but again that her resource constraint rules this out. She can only bid all her resources, which she does whenever $\lambda < \frac{1}{2}$. For any r she then ends up with the lower effective bid than player 2, because his best response is to generally bid half of his resources, and because his military potential is more than twice as high. From an ex ante perspective, the player with a comparative advantage in warfare consequently wins the war with higher probability than her opponent if and only if her military potential is at least half as good as her opponent's military potential. These results are illustrated in Fig. 2, where white regions indicate that player 1 is more likely to win, and gray regions that player 2 is more likely to win.

Following the discussion on which of the players is more likely to win from an ex ante perspective, we finally look at the players' relative expected military strengths, i.e., on the relative expected effective bids $\psi(R) \equiv \frac{\int_0^1 \lambda_1 f_1(r_1)\,dr_1}{\int_0^1 \lambda_2 f_2(r_2)\,dr_2} = \tilde{\lambda} R \frac{\int_0^1 f_1(r_1)\,dr_1}{\int_0^1 f_2(r_2)\,dr_2}$, and how they depend on the players' relative expected resource endowments $R \equiv \frac{R_1}{R_2}$. Fig. 6 illustrates the relationship between R and $\psi(R)$ for $\tilde{\beta} = 1$ and various values of $\tilde{\lambda}$.[17] We see that $\psi(0) = \frac{1}{2}$ for all values of $\tilde{\lambda}$. This result follows directly from Proposition 1, which implies that player 1 bids all her resources, and player 2 just enough to win with probability one, and from the uniform distributions of r_1 and r_2. More importantly, we find that $\psi(R)$ is increasing in R and $\tilde{\lambda}$. These results are intuitively appealing. They suggest that if a player becomes relatively better endowed in expectations, or relatively better in fighting, then his (or her) expected effective bid increases relative to the opponent's expected effective bid.[18]

[17] The derivation of $\psi(R)$, on which Fig. 6 is based, is available from the authors upon request.

[18] Further characteristics of $\psi(R)$ are related to the transitions between the different regions of the parameter space. The threshold value separating regions A and B is $R = (2\tilde{\lambda})^{-1}$ when $\tilde{\beta} = 1$. Fig. 2 shows that this threshold marks an inflection point: $\psi(R)$ is convex in region A, but concave in region B (given $\tilde{\lambda} > 1$). Therefore, the slope of $\psi(R)$ is generally highest at $R = (2\tilde{\lambda})^{-1}$, which implies that $\psi(R)$ is most sensitive to changes in R when both players are similarly likely to win from an ex ante perspective. The threshold value of R that separates regions B and C is also decreasing in $\tilde{\lambda}$; and $\psi(R)$

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5. Conclusions

We have presented a model of conflicts and wars in which the outcome of the war is uncertain from the countries' perspective because they lack information about their opponents' resources, and not because of luck on the battlefield as in the standard models of conflicts and wars. We have then characterized monotone continuous equilibrium strategies. We have seen that if the country with a comparative advantage in warfare has a large absolute disadvantage in warfare, then it allocates all resources to warfare, but is still unlikely to win the war against its much stronger opponent that only allocates some fraction of its resources to warfare. But if the country with a comparative advantage in warfare has also an absolute advantage or only a modest absolute disadvantage in warfare, then this country allocates a higher share of its resources to warfare at low realizations of the countries' resource levels, while absolute advantages matter at high realizations because no country wants to divert many more resources away from production when already winning the war with high probability. The country with a comparative advantage in warfare nevertheless tends to end up with the stronger military and is more likely to win the war.

There are noteworthy differences in the implications of our model and those of the standard models of conflicts and wars. Our model suggests that equilibrium play and the outcome of the war depend on absolute advantages in warfare, the distribution of resources between the rivalling countries, and the countries' expectations about their own and the opponent's resource endowment. In contrast, these factors play no crucial role in the standard models. We think that further studying the role of these factors in warfare is an interesting and promising avenue for future research, and we suggest considering our model as a more realistic alternative to the standard models for applied research on conflicts and wars.

Finally, we would like to highlight that the equilibrium strategies are similar in an alternative version of our game in which the winner receives the resources that the loser allocated to production (rather than the produced goods) and can then use these resources to produce consumption goods with its own production technology. In this alternative version, production technologies play no crucial role and the equilibrium strategies coincide with the equilibrium strategies in our model when $\beta = \frac{R_1}{R_2}$. This slightly simpler game could also represent contests in firms or political parties. In a firm, two groups may invest resources to convince the CEO that it is their product that should be developed and/or marketed, and the winning group can then use all remaining resources to develop, produce and market this product. In a political party, two politicians may collect campaign contributions in the primaries to become their party's candidate, and the winner can then exhaust the remaining contribution potential of all donors supporting this party in the main electoral race.

Appendix A

Proof of Lemma 2. The system of the differential equations (2) and (4), which is defined for $y \in A \subseteq [0, \min\{\frac{1}{\lambda}, 1\}]$, characterizes mutual best responses. The terms in the square brackets on the left-hand sides of (2) and (4) are the same, which implies $(\ln[f_2^{-1}(\lambda y)])' = \frac{\beta}{\lambda}(\ln[f_1^{-1}(y)])'$ and, consequently,

$$f_2^{-1}(\lambda y) = K_0 f_1^{-1}(y)^{\frac{\beta}{\lambda}}, \tag{16}$$

where K_0 is a constant. Substituting this expression into (2) and (4), we obtain two independent differential equations:

$$-\beta f_2^{-1}(\lambda y) + \left[\beta\left(K_0^{-\frac{\lambda}{\beta}} f_2^{-1}(\lambda y)^{\frac{\lambda}{\beta}} - y\right) + f_2^{-1}(\lambda y) - \lambda y\right]\frac{df_2^{-1}(\lambda y)}{dy} = 0, \tag{17}$$

$$-f_1^{-1}(y) + \left[\beta\left(f_1^{-1}(y) - y\right) + K_0 f_1^{-1}(y)^{\frac{\beta}{\lambda}} - \lambda y\right]\frac{df_1^{-1}(y)}{\lambda\, dy} = 0. \tag{18}$$

We rename the variable $y = \frac{z}{\lambda}$ in (17) to obtain

$$-\beta f_2^{-1}(z) + \left[\beta\left(K_0^{-\frac{\lambda}{\beta}} f_2^{-1}(z)^{\frac{\lambda}{\beta}} - \frac{z}{\lambda}\right) + f_2^{-1}(z) - z\right]\frac{\lambda\, df_2^{-1}(z)}{dz} = 0, \tag{19}$$

where $z \in \lambda A$. After rewriting (18) and (19) using $y = f_1(r_1)$ and $z = f_2(r_2)$, and rearranging terms, we get

$$\lambda\frac{df_1(r_1)}{dr_1} = \beta + K_0 r_1^{\frac{\beta}{\lambda}-1} - (\lambda+\beta)\frac{f_1(r_1)}{r_1}, \tag{20}$$

$$\beta\frac{df_2(r_2)}{dr_2} = \lambda + \lambda\beta K_0^{-\frac{\lambda}{\beta}} r_2^{\frac{\lambda}{\beta}-1} - (\lambda+\beta)\frac{f_2(r_2)}{r_2}. \tag{21}$$

Note that $r_1 \in f_1^{-1}(A)$ in (20) and $r_2 \in f_2^{-1}(\lambda A)$ in (21). Eqs. (5) and (6) are the solution to (20) and (21). □

always shows a kink at this value of R. The reason for this kink is that player 2 is constrained in region C (but not in B), and therefore not able to fight as aggressively as he would like to.

Proof of Lemma 4. Evaluate (16) at $y = c_l$ to get $K_0 = 2\lambda c_l^{1-\frac{\beta}{\lambda}}$. Then substitute K_0 into (5) and (6), evaluate (5) at $r_1 = c_l$ and (6) at $r_2 = 2\lambda c_l$, and use boundary condition (11) to get

$$K_1 = \left(\frac{\lambda}{\beta + 2\lambda} - \frac{\lambda}{2\beta + \lambda}\right) 2c_l^{2+\frac{\beta}{\lambda}}, \tag{22}$$

$$K_2 = \left(\frac{\lambda}{\beta + 2\lambda} - \frac{\lambda}{2\beta + \lambda}\right) (2\lambda c_l)^{2+\frac{\lambda}{\beta}}. \tag{23}$$

Then plug K_0, K_1 and K_2 into (5) and (6) to obtain (12) and (13).

It follows from the definition of $h(x)$ that $h'(x) > 0$, $h''(x) < 0$ and $h(1) = h'(1) = 1$; and from (12) and (13) that $f_1'(r_1) = h'(\frac{r_1}{c_l})$, $f_2'(r_2) = \frac{1}{2} h'((\frac{r_2}{2\lambda c_l})^{\frac{\lambda}{\beta}}) \frac{\lambda}{\beta} (\frac{r_2}{2\lambda c_l})^{\frac{\lambda-\beta}{\beta}}$ and $f_1''(r_1) = \frac{1}{c_l} h''(\frac{r_1}{c_l})$. Consequently, $f_1'(r_1) > 0$, $f_1'(c_l) = 1$, $f_1''(r_1) < 0$ and $f_2'(r_2) > 0$. □

Proof of Proposition 2. To avoid confusion, we denote the strategies described by (12) and (13) by $\tilde{f}_1(r_1)$ and $\tilde{f}_2(r_2)$, respectively.

First, we prove that $c_l < 1$ and $2\lambda c_l < 1$. Note that $2\lambda c_l = (\frac{1}{2\lambda})^{\frac{\beta}{\lambda-\beta}}$, where $\frac{1}{2\lambda} < 1$ and $\frac{\beta}{\lambda-\beta} > 0$ since $\max\{\frac{1}{2}, \beta\} < \lambda$. Hence $2\lambda c_l < 1$. It follows from $2\lambda c_l < 1$ and $\frac{1}{2} < \lambda$ that $c_l < 1$.

Second, we prove that $f_i(r_i) \leqslant r_i$ for all $r_i \in [0,1]$ and $i \in \{1,2\}$. It is straightforward that $f_1(r_1) = r_1 \leqslant r_1$ for all $r_1 \in [0, c_l]$, and it holds that $f_1(r_1) \leqslant r_1$ for all $r_1 \in (c_l, 1]$ since $\tilde{f}_1'(c_l) = 1$ and $\tilde{f}_1''(r_1) < 0$ for $r_1 > c_l$. It is also straightforward that $f_2(r_2) = \frac{r_2}{2} \leqslant r_2$ for all $r_2 \in [0, 2\lambda c_l]$. Hence we only need to identify the region in the parameter space in which $\tilde{f}_2(r_2) \leqslant r_2$ for all $r_2 \in (2\lambda c_l, 1]$ when $c_l = (2\lambda)^{\frac{\lambda}{\beta-\lambda}}$, or, equivalently, $h(\omega^{\frac{\lambda}{\beta}}) \leqslant 2\omega$ for all $\omega \equiv \frac{r_2}{2\lambda c_l} \in (1, \frac{1}{2\lambda c_l}] = (1, (2\lambda)^{\frac{\beta}{\lambda-\beta}}]$. Using the definition of $h(x)$, $h(\omega^{\frac{\lambda}{\beta}}) \leqslant 2\omega$ can be rewritten as

$$\frac{\beta}{\beta + 2\lambda} \omega^{\frac{\beta+2\lambda}{\beta}} - \frac{4\beta}{2\beta + \lambda} \omega^{\frac{2\beta+\lambda}{\beta}} \leqslant \frac{2\lambda(\lambda - \beta)}{(\beta + 2\lambda)(2\beta + \lambda)}. \tag{24}$$

The first derivative of the left-hand side of (24) is zero only when $\omega = 4^{\frac{\beta}{\lambda-\beta}}$, and the second derivative of the left-hand side evaluated at $\omega = 4^{\frac{\beta}{\lambda-\beta}}$ is strictly positive. Hence the left-hand side of (24) must be U-shaped with respect to ω. Thus, since the resource constraint is not violated at $r_2 = 2\lambda c_l$, we only need to verify that it is not violated at the top, i.e., at $r_2 = 1$. We therefore substitute $r_2 = 1$ and $c_l = (2\lambda)^{\frac{\lambda}{\beta-\lambda}}$ into $\tilde{f}_2(r_2) \leqslant r_2$ and rearrange to get $\lambda \leqslant \Lambda(\beta, \lambda)$.

Third, we prove that each player's equilibrium strategy is their global best response against their opponent's equilibrium strategy. We start with player 1. It directly follows from Lemma 3 that $f_1(r_1) = r_1$ is player 1's best response for $r_1 \leqslant c_l$. (Note that a deviation to some $y > c_l$ is not feasible in this case.) Now suppose $r_1 > c_l$. We know from Section 2 and Lemma 4 that $f_1(r_1) = \tilde{f}_1(r_1)$ is player 1's best response above c_l. Hence we only need to show that player 1 has no incentive to bid some $y \leqslant c_l$. When bidding some $y \leqslant c_l$, the payoff of player 1 would be $u_1(y; r_1) = \int_0^{2\lambda y} [\beta_1(r_1 - y) + \beta_2 \frac{r_2}{2}] \, dr_2$. The first derivative is

$$\frac{\partial u_1(y; r_1)}{\partial y} = \left[\beta_1(r_1 - y) + \beta_2 \lambda y\right] 2\lambda - \beta_1 2\lambda y = \frac{2\lambda}{\beta_2}\big(\beta(r_1 - y) + (\lambda - \beta) y\big), \tag{25}$$

and it must be positive since $\beta < \lambda$ and $y \leqslant c_l < r_1$. Hence player 1 has an incentive to increase his bid whenever $y \in [0, c_l]$ and $r_1 > c_l$.

We now turn to player 2. Given player 1's equilibrium strategy, the payoff of player 2 when bidding y is

$$u_2(y; r_2) = \begin{cases} \int_0^{\frac{y}{\lambda}} \beta_2(r_2 - y) \, dr_1 & \text{for } y \leqslant \min\{\lambda c_l, r_2\}, \\ \beta_1 \int_{c_l}^{\tilde{f}_1^{-1}(\frac{y}{\lambda})} (r_1 - \tilde{f}_1(r_1)) \, dr_1 + \beta_2 \int_0^{\tilde{f}_1^{-1}(\frac{y}{\lambda})} (r_2 - y) \, dr_1 & \text{for } \lambda c_l \leqslant y \leqslant r_2. \end{cases} \tag{26}$$

Suppose $r_2 \leqslant 2\lambda c_l$. We know from Lemma 3 that player 2's optimal bid less than $\min\{\lambda c_l, r_2\}$ is $y = \frac{r_2}{2}$. Hence we only need to show that player 1 has no incentive to bid some $y \in [\lambda c_l, r_2]$. For $y \in [\lambda c_l, r_2]$, it follows from (26) that

$$\frac{\partial u_2(y; r_2)}{\partial y} = \left[\beta_1\left(\tilde{f}_1^{-1}\left(\frac{y}{\lambda}\right) - \frac{y}{\lambda}\right) + \beta_2(r_2 - y)\right] \frac{d\tilde{f}_1^{-1}(\frac{y}{\lambda})}{dy} - \beta_2 \tilde{f}_1^{-1}\left(\frac{y}{\lambda}\right). \tag{27}$$

By construction of $\tilde{f}_2(r_2)$, this derivative is zero when $r_2 = \tilde{f}_2^{-1}(y)$. Since $r_2 \leqslant 2\lambda c_l \leqslant \tilde{f}_2^{-1}(y)$ and $\frac{d\tilde{f}_1^{-1}(\frac{y}{\lambda})}{dy} > 0$, $\frac{\partial u_2(y; r_2)}{\partial y}$ must be negative. Hence player 2 has an incentive to reduce his bid whenever $y \in [\lambda c_l, r_2]$ and $r_2 \leqslant 2\lambda c_l$. Now suppose $r_2 > 2\lambda c_l$. We know from Section 2 and Lemma 4 that $f_2(r_2) = \tilde{f}_2(r_2)$ is player 2's best response above λc_l. Hence we only need to show that player 1 has no incentive to bid some $y \leqslant \lambda c_l$. For $y \leqslant \lambda c_l$, it follows from (26) that

$$\frac{\partial u_2(y;r_2)}{\partial y} = \frac{\beta_2}{\lambda}(r_2 - 2y), \tag{28}$$

which must be positive since $r_2 \geqslant 2\lambda c_l$ and $y \leqslant \lambda c_l$. Hence player 2 has an incentive to increase his bid whenever $y \leqslant \lambda c_l$ and $r_2 > 2\lambda c_l$.

Finally, we prove that $\max\{\beta, \frac{1}{2}\} < \lambda \leqslant \Lambda(\beta, \lambda)$ implies $\Lambda(\beta, \lambda) > 2$. It can be shown that $\frac{h(x)}{x}$ is decreasing whenever $x > 1$. Hence $\Lambda(\beta, \lambda) = [\frac{h(x)}{x}]^{-1}$ increases as $x = (2\lambda)^{\frac{\lambda}{\lambda-\beta}} > 1$ increases. Since $\frac{\partial x}{\partial \lambda} > 0$ and $\frac{\partial x}{\partial \beta} > 0$ whenever $\lambda \geqslant \beta$, the chain rule implies that $\frac{\partial \Lambda(\beta,\lambda)}{\partial \lambda} = \frac{\partial \Lambda(\beta,\lambda)}{\partial x}\frac{\partial x}{\partial \lambda} > 0$ and $\frac{\partial \Lambda(\beta,\lambda)}{\partial \beta} = \frac{\partial \Lambda(\beta,\lambda)}{\partial x}\frac{\partial x}{\partial \beta} > 0$ whenever $\lambda \geqslant \beta$. Thus, in the set defined by $\lambda \leqslant \Lambda(\beta, \lambda)$, $\Lambda(\beta, \lambda)$ is smallest at the boundary characterized by $\lambda = \Lambda(\beta, \lambda)$. Now, we look for the point at which the level curve $\lambda = \Lambda(\beta, \lambda)$ intersects with $\lambda = \beta$. It can be shown that $\lim_{\lambda \to \beta^+} \Lambda(\beta, \lambda) = (\frac{1}{3} + \frac{1}{3\beta})^{-1} = \frac{3\lambda}{\lambda+1}$ since $\lambda > \frac{1}{2}$ and $\lambda \geqslant \beta$. At the intersection it must hold that $\lim_{\lambda \to \beta^+} \Lambda(\beta, \lambda) = \frac{3\lambda}{\lambda+1}$ and $\Lambda(\beta, \lambda) = \lambda$, which requires $\lambda = 2$. Therefore $\max\{\beta, \frac{1}{2}\} < \lambda \leqslant \Lambda(\beta, \lambda)$ implies $\Lambda(\beta, \lambda) > 2$. □

Proof of Proposition 3. We again denote the strategies described by (12) and (13) by $\tilde{f}_1(r_1)$ and $\tilde{f}_2(r_2)$, respectively.

First, we derive the thresholds c_l and c_h, and prove the uniqueness of c_l, $2\lambda c_l < 1$ and $c_l < c_h < 1$. Boundary condition (15) and Lemma 4 imply

$$\lambda c_l h\left(\frac{c_h}{c_l}\right) = \lambda c_l h\left(\left(\frac{1}{2\lambda c_l}\right)^{\frac{\lambda}{\beta}}\right) = 1. \tag{29}$$

Since $h(\cdot)$ is strictly increasing, the first equality implies $c_h = (2\lambda)^{-\frac{\lambda}{\beta}} c_l^{\frac{\beta-\lambda}{\beta}}$. The second equality gives the implicit definition of c_l. To prove existence and uniqueness of c_l, we rewrite this second equality as $x = \phi(x)$, where $x = 2\lambda c_l$ and $\phi(x) = 2(h(x^{-\frac{\lambda}{\beta}}))^{-1}$. Note that $\phi(\cdot)$ is not well-defined when $x = 0$, and that $\phi : (0, 1] \to (0, 2]$ is a continuous and increasing function with $\lim_{x \to 0^+} \phi(\cdot) = 0$ and $\phi(1) = 2$. Suppose condition (14) is violated and let $\varepsilon = (2\lambda)^{-\frac{\beta}{\lambda-\beta}} < 1$. Then it can be shown that $\phi(\varepsilon) < \varepsilon$. Hence $\phi(\cdot)$ has a fixed point $x^* \in (0, 1)$ satisfying $x^* = \phi(x^*)$ whenever condition (14) is violated. Moreover, this fixed point is unique since $\phi'(x^*) > 1$ whenever $x^* = \phi(x^*)$. Hence there exists a unique c_l, and it must hold that $2\lambda c_l < 1$. It follows from $2\lambda c_l < 1$ that $1 < (2\lambda c_l)^{-\frac{\lambda}{\beta}}$ and, consequently, $c_l < c_h$; and it follows from $\lambda > \max\{\frac{1}{2}, \beta\}$ that $c_l < (2\lambda c_l)^{\frac{\lambda}{\beta}}$ and, consequently, $c_2 < 1$.

Second, we prove that $f_i(r_i) \leqslant r_i$ for all $r_i \in [0, 1]$ and $i \in \{1, 2\}$. It is straightforward that $f_1(r_1) = r_1 \leqslant r_1$ for all $r_1 \in [0, c_l]$, and it holds that $f_1(r_1) \leqslant r_1$ for all $r_1 \in (c_l, 1]$ since $\tilde{f}_1'(c_l) = 1$, $\tilde{f}_1''(r_1) < 0$ for $r_1 \in (c_l, c_h]$ and $\tilde{f}_1(r_1) = \tilde{f}_1(c_h)$ for $r_1 \in (c_h, 1]$. It is also straightforward that $f_2(r_2) = \frac{r_2}{2} \leqslant r_2$ for all $r_2 \in [0, 2\lambda c_l]$; and c_l is chosen such that player 2's resource constraint is binding when $r_2 = 1$. It can be shown along the same lines as in the proof of that player 2 also bids strictly less then his resources for $r_2 \in (2\lambda c_l, 1)$.

Third, we prove that each player's equilibrium strategy is their global best response against their opponent's equilibrium strategy. The corresponding part of the proof of Proposition 2 applies here as well, as the arguments do not assume a value for c_l. Hence we only need to show that player 1 has no incentive to deviate for $r_1 \in (c_h, 1]$. Therefore, suppose $r_1 \in (c_h, 1]$ and consider a bid $y \in [c_l, \frac{1}{\lambda}]$. Differentiating player 1's payoff with respect to y then yields

$$\frac{\partial u_1(y;r_1)}{\partial y} = \left[\beta_1(r_1 - y) + \beta_2\left(\tilde{f}_2^{-1}(\lambda y) - \lambda y\right)\right]\frac{d\tilde{f}_2^{-1}(\lambda y)}{dy} - \beta_1 \tilde{f}_2^{-1}(\lambda y). \tag{30}$$

By construction of $\tilde{f}_1(r_1)$, this derivative is zero when $r_1 = \tilde{f}_1^{-1}(y) \leqslant c_h$. Thus, since $\frac{d\tilde{f}_2^{-1}(\lambda y)}{dy} > 0$, $\frac{\partial u_1(y,r_1)}{\partial y}$ must be positive when $r_1 \geqslant c_h$, implying that in this case player 1 can profitably increase his bid y. □

Proof of Proposition 5. We first prove the last statement comparing effective bids. It directly follows from the equilibrium strategies described in Proposition 1 that $\lambda f_1(r) < f_2(r)$ for all $r \in (0, 1)$ if $\lambda < \frac{1}{2}$, and that $\lambda f_1(r) = f_2(r)$ for all $r \in (0, 1)$ if $\lambda = \frac{1}{2}$. To prove that $\lambda f_1(r) > f_2(r)$ for all $r \in (0, 1)$ if $\lambda > \frac{1}{2}$, we first consider the case in which $\frac{1}{2} < \lambda \leqslant \Lambda(\beta, \lambda)$. Proposition 2 characterizes the equilibrium strategies for this case. Consider a particular $\tilde{y} \in A$ such that $\tilde{y} = \lambda f_1(r_1) = f_2(r_2)$. We need to show that $r_2 > r_1$. For $\tilde{y} \leqslant \lambda c_l$, it follows from $f_1(r_1) = r_1$ for $r_1 \in [0, c_l]$, $f_2(r_2) = \frac{r_2}{2}$ for $r_2 \in [0, 2\lambda c_l]$, and $\lambda > \frac{1}{2}$ that $r_2 \geqslant r_1$ must hold. For $\tilde{y} > \lambda c_l$, it follows from (12) and (13) and $h'(x) > 0$ that $\lambda f_1(r_1) = f_2(r_2)$ requires $r_1 = c_l(\frac{r_2}{2\lambda c_l})^{\frac{\lambda}{\beta}} = r_2^{\frac{\lambda}{\beta}}$, where the second equality follows from $c_l = (2\lambda)^{\frac{\lambda}{\beta-\lambda}}$. Since $\beta < \lambda$ and $r_i \in (0, 1)$ for $i = 1, 2$, $r_1 = r_2^{\frac{\lambda}{\beta}}$ implies $r_2 > r_1$. Hence $\lambda f_1(r) > f_2(r)$ for all $r \in (0, 1)$ if $\frac{1}{2} < \lambda \leqslant \Lambda(\beta, \lambda)$. It remains to consider the case in which $\lambda > \Lambda(\beta, \lambda)$. Proposition 3 characterizes the equilibrium strategies for this case. Using the same strategy as above, we can prove that $\lambda f_1(r) > f_2(r)$ for all $r \in (0, c_h)$. Moreover, it directly follows from $f_1(r_1) = \frac{1}{\lambda}$ for $r_1 \geqslant c_h$ and $f_2(r_2) \leqslant r_2$ that $\lambda f_1(r) > f_2(r)$ must also hold for all $r \in [c_h, 1)$.

We next prove the two statements comparing real bids. For $\lambda \leqslant \frac{1}{2}$, it directly follows from the equilibrium strategies described in Proposition 1 that $f_1(r) > f_2(r)$ for all $r \in (0, 1]$. We have shown above that $\lambda f_1(r) > f_2(r)$ for all $r \in (0, 1)$ if

$\lambda > \frac{1}{2}$. Hence it must hold that $f_1(r) > f_2(r)$ for all $r \in (0,1)$ if $\lambda \in (\frac{1}{2}, 1]$. For $\lambda > 1$, Propositions 2 and 3 imply $f_1(r) > f_2(r)$ for $r \in (0, 2\lambda c_l]$. Further it follows from Lemma 1 that $f_1(1) < f_2(1)$ if $\lambda > 1$. Hence the continuity of $f_1(r_1)$ and $f_2(r_2)$ and the intermediate value theorem imply that there must exists an odd number of thresholds $\hat{r}$ in the interval $(2\lambda c_l, 1)$ that satisfy $f_1(\hat{r}) = f_2(\hat{r})$. It holds that $f_1(r) > f_2(r)$ for all r below the lowest threshold and $f_1(r) < f_2(r)$ for all r above the highest threshold. □

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